Heteroscedastic Von Bertalanffy Growth Model and an Application to a Kubbard female chicken corporeal weight growth data

Carlos Alberto Ribeiro Diniz
DEs, Universidade Federal de S˜ ao Carlos, SP, Brazil, dcad@ufscar.br

Lia Hanna Martins Morita
Departamento de Matematica - UFMT, Cuñaba - MT, Brazil, lia@ufmt.br

Francisco Louzada-Neto
DEs, Universidade Federal de S˜ ao Carlos, SP, Brazil, dfln@ufscar.br

Abstract
In this paper we propose a heteroscedastic Von Bertalanffy growth model considering a heteroscedastic dispersion matrix. Maximum likelihood estimation procedure is addressed. The methodology is illustrated on a real Kubbard female chicken corporeal weight dataset.

Keyword: Growth Models, Multiplicative Heteroscedasticity, Maximum Likelihood.

1 Introduction
Sigmoidal growth models have been widely used for modeling animals and plant growths. Khamis et al. (2005) present twelve nonlinear growth models for oil palm yield growth. Ersoy et al. (2006), establish growth curves parameters of the American bronze turkeys. Sengul et al. (2005), use four different non-linear models to define growth curves of large white turkey. Among the S-shaped pattern models which are used to explain animals and plants growth we can mention the Brody, Richards, Gompertz, logistic, Von Bertalanffy, Weibull and Morgan-Mercer-Flodin growth models.

Usually, inferences on the parameter models are based on the classical approach, which consist of obtaining estimators via least squares or maximum likelihood methods on a homoscedastic error asymptotic normal distribution.

Although assuming homocedasticity leads, at least in principle, to a statistically treatable procedure. Presence of heterocedasticity in growth datasets is not uncommon in practice. For instance, consider the dataset composed by measures of corporeal weights of 13 Kubbard female chicken, fed with a comercial diet at the Empresa Brasileira de Pesquisa Agropecuária (Freitas, 2005). The birds were identified by an numbered aluminum ring placed in their right wings. All birds were weekly weighted for a period of seven weeks. The evaluations were always done at the same time and weekday. The bird average weekly are shown in the Table 1 where we also find their respective standard deviations. We observe that the weekly standard deviations increase with time. However, fitting one of those models described above, by considering the standard classical approach, we should assume that the error terms are identically distributed, which, definitively is not the Kubbard female chicken data case.

Table 1: Corporeal weights (in grams) with the average weekly bird weights and their standard deviations.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>133,310</td>
<td>321,380</td>
<td>559,920</td>
<td>807,540</td>
<td>1089,850</td>
<td>1473,000</td>
<td>1770,000</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2,800</td>
<td>6,290</td>
<td>15,390</td>
<td>19,690</td>
<td>14,880</td>
<td>20,880</td>
<td>22,950</td>
</tr>
</tbody>
</table>

In order to accommodate the presence of heteroscedasticity in growth datasets, in this paper we propose a heteroscedastic sigmoidal growth model, which the deterministic component is given by
The Von Bertalanffy model (Von Bertalanffy, 1938). The main idea is to consider a heteroscedastic multiplicative error in the modelling. The choice of the Von Bertalanffy sigmoidal growth model is partly based on its interpretative appeal. This model was proposed as a mechanistic model for animal growth, considering the difference between the metabolic forces of anabolism and catabolism. However, our approach is general and, in principle, may be extended to others sigmoidal growth models.

The paper is organized as follows. Section 2 presents the model formulation. The likelihood function, starting values for the parameters in the model, the asymptotic covariance matrix and model comparison are presented in Section 3. In Section 4 our methodology is illustrated on the real female chicken data set presented above. Final comments in Section 5 conclude the paper.

2 Model Formulation

The well known Von Bertalanffy model is given by (von Bertalanffy, 1938),

$$y_i = \alpha(1 - \beta \exp(-\gamma x_i))^3 + \varepsilon_i,$$

where \(\alpha\) is the asymptote of the curve, \(\beta\) determines the intercept, \(\gamma\) determines the growth rate and \(\varepsilon\) is a vector \(n \times 1\) of i.i.d. normal errors with mean zero and specified variance-covariance matrix. Following Lester, Shute and Abrams (2004) the von Bertalanffy model (1) is a three parameters sigmoidal growth model which provides a good description of somatic growth after animal maturity. Its parameters are simple functions of age at maturity and reproductive investment.

Usually, the inference on the parameters is based on a homoscedastic error term model. In this case, \(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n\) are independent and identically distributed normal with mean zero and unknow variance \(\sigma^2\). However, when there is a need of explaining the phenomenon of growth in the presence of heterogeneity, it is required to introduce a heteroscedastic structure for the error terms in the model.

In this paper we consider the multiplicative heteroscedasticity model discussed by Harvey (1976). This choice of heteroscedasticity consists in express the variance \(\sigma^2\) in the form

$$\sigma_i^2 = \sigma^2 x_i^\lambda,$$

where \(\lambda\) is an unknown parameter which determines the degree of heteroscedasticity. Although we may express the heteroscedasticity in different ways, the form (2) seems to be a natural choice, since it represents a log-linear relationship between \(\log(\sigma_i^2)\) and \(\log(x_i)\), with intercept parameter equals to \(\log(\sigma^2)\) and slope parameter equals to \(\lambda\), which seems to be the case for the Kubbard female chicken data set. Also, Figure 1 shows the residuals for the Kubbard female chicken data when a homoscedastic Von Bertalanffy model was fitted to the data. The megaphone-shaped pattern is a characteristic of a heteroscedastic model. In fact, such pattern indicates that the variances of the error terms tend to increase as times \(x_i\)'s increase, which justify the multiplicative error in the modelling (2), which was assumed here. The Goldfeld-Quandt statistic is \(F[36,36] = 4.169\) with p-value < 0.000, which is a strong evidence in favour of the hypothesis of heteroscedastic.

Under the conditions stated above, the error terms \(\varepsilon_i\) are assumed to be

$$\varepsilon_i \sim N \left(0, \sigma^2 x_i^\lambda \right).$$

3 Inference

For inference we adopt a naive maximum likelihood approach. The likelihood function, starting values for the parameters in the model, details of the asymptotic covariance matrix and model comparison are described below.

The likelihood function of \(\theta\) and \(\sigma\), where \(\theta = (\alpha, \beta, \gamma, \lambda)\) for model (1), given the sample vectors \(x = (x_1, x_2, \ldots, x_n)'\) and \(y = (y_1, y_2, \ldots, y_n)'\), is obtained by the product of the error density functions, that is,

$$L (\theta, \sigma | x, y) = (2\pi \sigma^2)^{-n/2} |\Sigma|^{-1/2} \exp \left\{ -\frac{1}{2\sigma^2} [f(x, \theta) - y]' \Sigma^{-1} [f(x, \theta) - y] \right\},$$

where \(f(x, \theta) = \alpha(1 - \beta \exp(-\gamma x))^3\).
where \((f(x, \theta) - y) = (f(x_1, \theta) - y_1, f(x_2, \theta) - y_2, \ldots, f(x_n, \theta) - y_n)\) and \([f(x, \theta) - y]' \Sigma^{-1} [f(x, \theta) - y] = \sum_{i=1}^{n} x_i^{-\lambda} |y_i - f(x_i, \theta)|^2.\)

Thus, the log-likelihood function can be written as

\[
\ln[L(\theta, \sigma | x, y)] = -\frac{n}{2} \ln(2\pi\sigma^2) - \frac{1}{2} \sum_{i=1}^{n} \ln(x_i) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} x_i^{-\lambda} |y_i - f(x_i, \theta)|^2, \tag{5}
\]

where \(f(x_i, \theta)\) is given by (1). The maximum likelihood estimators can be obtained by solving the system of nonlinear equations given by the partial derivatives of (5) with respect to the parameters, which are given by

\[
\frac{d}{d\alpha} \ln[L(\theta, \sigma | x, y)] = \sum_{i=1}^{n} x_i^{-\lambda}(y_i - \alpha(1 - \beta \exp(-\gamma x_i))^3)(1 - \beta \exp(-\gamma x_i))^3 / \sigma^2,
\]

\[
\frac{d}{d\beta} \ln[L(\theta, \sigma | x, y)] = \sum_{i=1}^{n} 3x_i^{-\lambda}(y_i - \alpha(1 - \beta \exp(-\gamma x_i))^3)\alpha(1 - \beta \exp(-\gamma x_i))^2 \exp(-\gamma x_i) / \sigma^2,
\]

\[
\frac{d}{d\gamma} \ln[L(\theta, \sigma | x, y)] = \sum_{i=1}^{n} -3x_i^{-\lambda}(y_i - \alpha(1 - \beta \exp(-\gamma x_i))^3)\alpha(1 - \beta \exp(-\gamma x_i))^2 \beta x_i \exp(-\gamma x_i) / \sigma^2,
\]

\[
\frac{d}{d\sigma^2} \ln[L(\theta, \sigma | x, y)] = \sum_{i=1}^{n} -x_i^{-\lambda}(y_i - \alpha(1 - \beta \exp(-\gamma x_i))^3)^2 / 2\sigma^2,
\]

\[
\frac{d}{d\sigma} \ln[L(\theta, \sigma | x, y)] = \sum_{i=1}^{n} -x_i^{-\lambda}(y_i - \alpha(1 - \beta \exp(-\gamma x_i))^3)^2 / 2\sigma^4.
\]

We considered here the Quasi-Newton method (Dennis and Schmabel, 1983) as our optimization algorithm. The algorithm needs the starting parameter values. Following Ratkowsky (1983), the choice of the starting values for the model (1) parameters are defined via the following linearization

\[
z_i = \log \left(1 - \left(\frac{y_i}{\alpha^{(0)}}\right)^{\frac{1}{\beta}}\right) = \log(\beta) - \gamma x_i,
\]

where \(\alpha^{(0)}\) is chosen through visual inspection of the data. The starting values for \(\beta\) and \(\gamma\) in the model (1) are given by

\[
\gamma^{(0)} = \frac{z_i - z_j}{x_j - x_i} \quad \text{and} \quad \beta^{(0)} = \exp\left(z_i + \gamma^{(0)} x_i\right), \quad \text{where} \ i \neq j.
\]

Large sample inference for the parameters can be based, in principle, on the MLEs and their estimated standard errors. Assuming an asymptotic normal distribution for the MLEs (Cox and Hinkley, 1974), with the estimated asymptotic covariance matrix of \((\hat{\theta}, \hat{\sigma})\) obtained through the inverse of the observed Fisher information matrix, given by

\[
I(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\lambda}, \hat{\sigma}) = \frac{1}{\sigma^2} \begin{bmatrix}
    b_1 & -3b_2 & 3b_3 & b_1 & b_5/\hat{\sigma}^2 \\
    -3b_2 & \hat{\alpha}b_6 & -3\hat{\alpha}b_7 & -3\hat{\alpha}b_8 & -3\hat{\alpha}^2 b_7/\hat{\sigma}^2 \\
    3b_3 & -3\hat{\alpha}b_7 & 3\hat{\alpha}\hat{\beta}b_{10} & 3\hat{\alpha}\hat{\beta}b_{11} & 3\hat{\alpha}\hat{\beta}b_{12}/\hat{\sigma}^2 \\
    b_4 & -3\hat{\alpha}b_8 & 3\hat{\alpha}\hat{\beta}b_{11} & -0.5b_{13} & 0.5b_{14}/\hat{\sigma}^2 \\
    b_5/\hat{\sigma}^2 & -3\hat{\alpha}^2 b_{7}/\hat{\sigma}^2 & 3\hat{\alpha}\hat{\beta}b_{12}/\hat{\sigma}^2 & 0.5b_{14}/\hat{\sigma}^2 & b_{15}/\hat{\sigma}^4
\end{bmatrix}, \tag{7}
\]

where \(b_1\) to \(b_{15}\) are given in Appendix A.

Model selection is a very important issue. In the literature, there are several methodologies that intend to analyze the adequability of a model besides selecting the best fit among a collection of models. For instance, among several existent techniques (Paulino, 2003), we can compute the
Table 2: The MLEs and 95% asymptotical confidence intervals (CI), for α, the coordinates of the inflection point (X_F, Y_F), σ^2 and λ for the model fitting. Also, the l(\hat{\theta}_j), AIC and BIC values for the model in its homocedastic and heteroscedastic versions.

<table>
<thead>
<tr>
<th>λ</th>
<th>Parameter</th>
<th>MLE</th>
<th>95% CI</th>
<th>l(\hat{\theta}_j)</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>α</td>
<td>5940.06</td>
<td>[5939.36; 5940.76]</td>
<td>-499.69</td>
<td>1007.38</td>
<td>1009.64</td>
</tr>
<tr>
<td></td>
<td>X_F</td>
<td>6.95</td>
<td>[6.87; 7.02]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Y_F</td>
<td>1760.02</td>
<td>[1759.81; 1760.22]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>σ^2</td>
<td>3519.61</td>
<td>[2483.17; 4556.06]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>≠ 0</td>
<td>α</td>
<td>4676.25</td>
<td>[3668.43; 5684.06]</td>
<td>-480.50</td>
<td>970.99</td>
<td>973.79</td>
</tr>
<tr>
<td></td>
<td>X_F</td>
<td>5.81</td>
<td>[4.90; 6.72]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Y_F</td>
<td>1385.56</td>
<td>[1090.91; 1680.20]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>σ^2</td>
<td>152.87</td>
<td>[34.24; 271.51]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>λ</td>
<td>2.20</td>
<td>[1.61; 2.79]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

maximum values of the log-likelihoods, say \( l(\hat{\theta}_j) \), under the model \( j \) and calculate the likelihood ratio statistics, LRS, defined by \( -2(l(\hat{\theta}_{M_1}) - l(\hat{\theta}_{M_0})) \) where \( M_0 \) and \( M_1 \) are two nested models, with an asymptotic chi-squared distribution with 1 degree of freedom. Also, we can calculate the Akaike information criterion (AIC) and Bayesian information criterion (BIC), which are defined, respectively, by \( -2l(\hat{\theta}_j) + 2p \) and \( -2l(\hat{\theta}_j) + p \log(n) \), where \( \hat{\theta}_j \) is the maximum likelihood estimate under model \( j \), \( p \) is the number of parameters estimated under model \( j \), and \( n \) is the sample size. Best model correspond to a high LRS and lower AIC and BIC values.

4 Kubbard Female Chicken Data

In this section our methodology is illustrated on the real female chicken data set presented in Section 1. The Von Bertalanffy model (1) in its homoscedastic and heteroscedastic versions, were fitted to the data.

Table 2 presents the MLEs, together with their 95% asymptotical confidence intervals, for the asymptote \( \alpha \), the coordinates of the inflection point \( (X_F, Y_F) \), the variance \( \sigma^2 \) and the heteroscedasticity parameter \( \lambda \). The coordinates of the inflection point \( (X_F, Y_F) \) have an important biology interpretability (Seber and Wild, 1989). The inflection point is the moment in which the animal switch from the progressive growth phase to regressive one, that is, the growth rate begins to decrease due to many factors which inhibit progressively the growth. We first note that the homoscedasticity assumption is not satisfied for this dataset since the confidence intervals for the heteroscedasticity parameter \( \lambda \) do not include the zero value. This result is corroborated by the \( l(\hat{\theta}_j) \) values and the AIC and BIC statistics values. The LRS for testing the homoscedasticity Von Bertalanffy model against the heteroscedastic model is equals to 38.38 with a \( p \)-value smaller than 0.00001 by considering an asymptotic chi-squared distribution with 1 degree of freedom. These results provide strong evidence in favour of the heteroscedastic Von Bertalanffy model, which presents a better fit than its particular case to the data set under consideration.

The MLE of the asymptote \( \alpha \) and the coordinates \( (X_F, Y_F) \) are larger in the model in its homoscedasticity version than in its heteroscedasticity version (Table 2). Overall, the model in its heteroscedasticity version seems to be appealing from the practical point of view, while keeping coherence on the MLEs obtained values. The model in its homoscedastic version considers the chickens should be abated by 6.9 weeks (the coordinate \( X_F \) in Table 2), while the model in its heteroscedastic version considers the chickens should be abated much earlier by 5.8 weeks. A time economy of approximately 16%.

Figure 2 shows the sampling corporeal weights for a period of seven weeks, with the homoscedasticity (left panel) and heteroscedastic (right panel) Von Bertalanffy model fitting (—) with the 95% lower and upper limit (⁻⁻⁻⁻⁻⁻) confidence bounds. Denying the presence of heteroscedastic leads to a false impression of a smaller confidence interval amplitude.
Figure 1: Left panel: Sample corporeal weights for a period of seven weeks, the homoscedasticity Von Bertalanffy model fitting (—) with the 95% lower and upper limit (----) confidence bounds. Right panel: Sample corporeal weights for a period of seven weeks, the heteroscedastic Von Bertalanffy model fitting (—) with the 95% lower and upper limit (----) confidence bounds.

5 Conclusions

In this paper we present the known sigmoidal growth model, namely, Von Bertalanffy model in its heteroscedastic version by considering a multiplicative heteroscedastic structure. We apply the methodology to a real data set on corporeal weights of Kubbard female chicken. We observed that the amplitude of the predictive confidence intervals are severely affected by the presence of heteroscedasticity in the data as the chicken age increases. However, an economical vantage comes out by considering it. Although we focus our study on the Von Bertalanffy model with a multiplicative heteroscedastic structure our methodology is general and, in principle, may be extended to others sigmoidal growth models with others heteroscedastic structures. This would however introduce extra difficulties in the analysis but needs further work. Also, only large sample inference was considered for the model parameters and hypothesis tests. We however know that such procedures may not perform well in presence of small samples and should be investigated further. A possibility is to consider the so called bootstrap scheme (Davison and Hinkley, 1997).

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References


Appendix A

The $b_1$ to $b_{15}$ values for the observed Fisher information matrix (7) are given above.

\[
b_1 = \sum_{i=1}^{n} x_i^{-\lambda}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^6,
\]

\[
b_2 = \sum_{i=1}^{n} x_i^{-\lambda}[\hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^5 - (y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2]\exp(-\hat{\gamma}x_i)
\]

\[
b_3 = \sum_{i=1}^{n} x_i^{-\lambda}[\hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^5 - (y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2]\hat{\beta}x_i\exp(-\hat{\gamma}x_i)
\]

\[
b_4 = \sum_{i=1}^{n} x_i^{-\lambda}\ln(x_i)(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3
\]

\[
b_5 = \sum_{i=1}^{n} x_i^{-\lambda}(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3
\]

\[
b_6 = \sum_{i=1}^{n} -6x_i^{-\lambda}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))\exp(-\hat{\gamma}x_i)^2[(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3 - (9/6)\hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3
\]

\[
b_7 = \sum_{i=1}^{n} x_i^{-\lambda}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))x_i\exp(-\hat{\gamma}x_i)[(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)
\]

\[
[(1 - \hat{\beta}\exp(-\hat{\gamma}x_i)) - 2\hat{\beta}\exp(-\hat{\gamma}x_i)] + 3\hat{\alpha}\hat{\beta}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2\exp(-\hat{\gamma}x_i)]
\]

\[
b_8 = \sum_{i=1}^{n} x_i^{-\lambda}\ln(x_i)[y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3](1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2\exp(-\hat{\gamma}x_i)
\]

\[
b_9 = \sum_{i=1}^{n} x_i^{-\lambda}(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2)\exp(-\hat{\gamma}x_i)
\]

\[
b_{10} = \sum_{i=1}^{n} x_i^{-\lambda}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))x_i^2\exp(-\hat{\gamma}x_i)\{(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)
\]

\[
[(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2 - 2\hat{\beta}\exp(-\hat{\gamma}x_i)] + 3\hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2\hat{\beta}\exp(-\hat{\gamma}x_i)]
\]

\[
b_{11} = \sum_{i=1}^{n} x_i^{-\lambda}\ln(x_i)(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2x_i\exp(-\hat{\gamma}x_i)
\]

\[
b_{12} = \sum_{i=1}^{n} x_i^{-\lambda}(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^2x_i\exp(-\hat{\gamma}x_i)
\]

\[
b_{13} = \sum_{i=1}^{n} x_i^{-\lambda}\ln(x_i)(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)^2
\]

\[
b_{14} = \sum_{i=1}^{n} x_i^{-\lambda}\ln(x_i)(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)^2
\]

\[
b_{15} = \sum_{i=1}^{n} x_i^{-\lambda}(y_i - \hat{\alpha}(1 - \hat{\beta}\exp(-\hat{\gamma}x_i))^3)^2
\]
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